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Frictional Self-Oscillation of a Deformable Washer Interacting with a Rigid Rod that Rotates at a Constant Angular Velocity

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Abstract

This article explores self-oscillations in a round washer interacting with a rotating elastic rod. It demonstrates that natural frequencies of the system are not equidistant. The internal friction in the washer material described by the Voigt model, limits the frequency range of excited oscillations. Quasi-harmonic self-oscillations with a limited range are excited in the washer due to the friction motion. Therefore, fluctuations apply to processes in the bearing outer race of a machine in the rotary mode.

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Keywords: frictional self-oscillations; rotation; deformable washer; rigid rod.

1. Problem statement

Inside a homogeneous round washer with an inside diameter R_1 and outside diameter R_2, a rigid screw is rotating at a constant angular velocity ω. The outside diameter of the wash is rigidly fixed (Fig. 1a).

This article explores deformation processes in the wash within plane elasticity, which in polar coordinates (r, θ) is described by the following set of equations [1]

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$$\begin{cases} \rho \dfrac{\partial^2 u_r}{\partial t^2} = \dfrac{\partial \sigma_r}{\partial r} + \dfrac{\sigma_r - \sigma_\theta}{r} + \dfrac{1}{r}\dfrac{\partial \tau_{r\theta}}{\partial \theta} \\ \rho \dfrac{\partial^2 u_\theta}{\partial t^2} = \dfrac{1}{r}\dfrac{\partial \sigma_\theta}{\partial \theta} + \dfrac{\partial \tau_{r\theta}}{\partial r} + \dfrac{2\tau_{r\theta}}{r} \end{cases}, \quad (1)$$

where ρ is the density of the wash material; $u_r(\theta, r, t)$ and $u_\theta(\theta, r, t)$ are the radial and circumferential offsets respectively (Fig. 1b); σ_r and σ_θ are normal stresses; $\tau_{r\theta}$ is shearing stress.

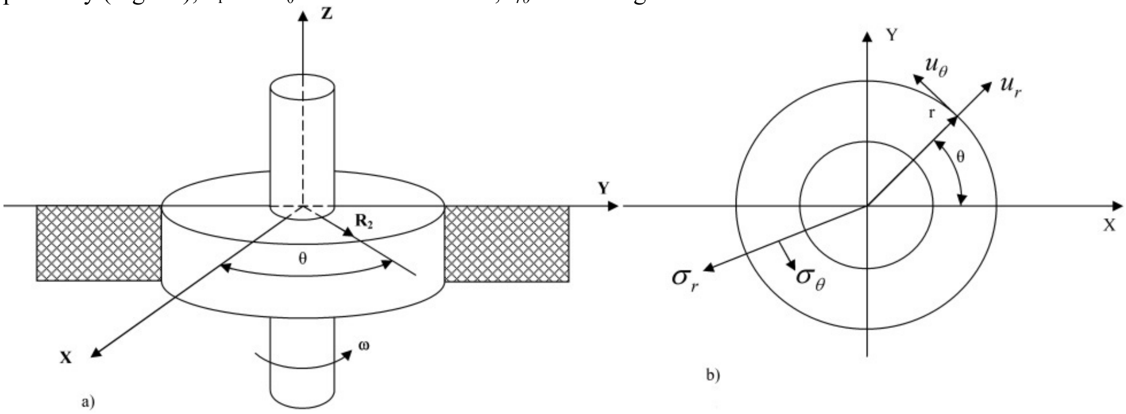

Fig. 1. a) Layout of the problem elements, b) Stress and offset paths.

Stress, deformation and displacements are connected by the following correlations

$$\begin{cases} \sigma_r^{(1)} = \left(\varepsilon_r + \nu\varepsilon_\theta\right)\dfrac{E}{1-\nu^2} \\ \tau_{r\theta}^{(1)} = \mu\gamma_{r\theta} \\ \varepsilon_r = \partial u_r / \partial r \\ \varepsilon_\theta = \dfrac{u_r}{r} + \dfrac{1}{r}\dfrac{\partial u_\theta}{\partial \theta} \\ \gamma_{r\theta} = 2\varepsilon_{r\theta} = \dfrac{1}{r}\dfrac{\partial u_r}{\partial \theta} + \dfrac{\partial u_\theta}{\partial r} - \dfrac{u_\theta}{r} \end{cases}, \quad (2)$$

where E is Young's modulus of elasticity; μ and ν are elastic constants; and index *1* is the elastic component of stress.

Total stress in viscoelastic material can be presented as a sum of elastic $\sigma_{ij}^{(1)}$ and dissipative $\sigma_{ij}^{(\alpha)}$ stress components [2]

$$\begin{aligned} \sigma_{ij} &= \sigma_{ij}^{(1)} + \sigma_{ij}^{(\alpha)} \\ \tau_{r\theta} &= \tau_{r\theta}^{(1)} + \tau_{r\theta}^{(\alpha)} \end{aligned}. \quad (3)$$

For Voigt viscoelastic material, dissipative components of the stress tensor are presented as derivatives of σ_i and $\tau_{r\theta}$ as shown below

$$\begin{aligned}\sigma_i^{\alpha} &= \eta \frac{\partial}{\partial t}\sigma_1^{(1)} \\ \tau_{r\theta}^{\alpha} &= \eta \frac{\partial}{\partial t}\tau_{r\theta}^{(1)}\end{aligned} . \quad (4)$$

Assume that the outside diameter ($2R_2$) of the wash is rigidly fixed, i.e.

$$u_r \Big|_{r=R_2} = 0, \quad (5)$$

$$u_\theta \Big|_{r=R_2} = 0, \quad (6)$$

whereas the inside diameter ($2R_1$) clamps the screw

$$\sigma_r \Big|_{r=R_1} = 0, \quad (7)$$

where σ_0 is the clamping stress.

A moment of dry friction M_{fr} acts between the screw and the wash, which depends on differential slip velocity of the contacting surfaces of the screw ($V_b=\omega R_1 t$) and the wash ($V=\partial u_\theta/\partial t$)

$$\left[\tau_{r\theta} = -M_{fr}\left(\omega R_1 t - \frac{\partial u_\theta}{\partial t}\right)\right]_{r=R_1} . \quad (8)$$

Dependence of M_{fr} on the slip velocity of contacting surfaces has the same "model" nature as detailed in the work of A.A.Vitt [3].

Axial symmetry of the wash suggests independence of displacement components (u_θ, u_r) from the angular coordinate θ. In the radial direction the processes are assumed quasi-static. As a result, the radial displacement component u_r, is a function of coordinate r only (that is $\partial(u_r)/\partial dt \equiv 0$). In view of these assumptions, the displacement boundary value problem (1)–(8) is split into two separate problems. One of them is static [4]

$$\frac{\partial^2 u_r}{\partial r^2} + \frac{\partial}{\partial r}\left(\frac{u_r}{r}\right) = 0, \quad (9)$$

$$u_r \Big|_{r=R_2} = 0, \quad (10)$$

$$\frac{E}{1-\nu^2}\left(\frac{\partial u_r}{\partial r} + \nu\frac{u_r}{r}\right)\Big|_{r=R_1} = \sigma_0 . \quad (11)$$

It describes radial stress deformations in the wash. The other problem is dynamic,

$$\frac{\partial^2 u_\theta}{\partial t^2} = \frac{\mu}{\rho}\frac{\partial}{\partial r}\left(\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\right) + \eta\frac{\partial^2}{\partial r \partial t}\left(\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\right), \quad (12)$$

$$u_\theta \Big|_{r=R_2} = 0, \quad (13)$$

$$\left[\mu\left(\frac{\partial u_\theta}{\partial r}-\frac{u_\theta}{r}\right)=-M_{TP}\left(\omega R_1 t-\frac{\partial u_\theta}{\partial t}\right)\right]_{r=R_1}. \tag{14}$$

It describes shear oscillations of the wash [5] and the generation of shear waves. Basic laws of propagation of shear waves in mechanical systems have been studied in [6-16].

2. Solution of the static problem

Problem (9)–(11) is similar to Lamé problem about finding the stress state of a cylinder under external and internal pressure [4]. The solution looks as follows:

$$u_r(r)=\frac{\sigma_0\left(1-\nu^2\right)R_1^2}{E\left[(\nu-1)-(1+\nu)\left(R_1/R_2\right)^2\right]}\frac{1-\left(r/R_2\right)^2}{r}, \tag{15}$$

where $R_1 \le r \le R_2$.

As the physical stress components (σ_r phys = σ_r; σ_r phys), then

$$\sigma_r=\sigma_0\frac{(\nu-1)-(1+\nu)\left(r/R_2\right)^2}{(\nu-1)-(1+\nu)\left(R_1/R_2\right)^2}, \tag{16}$$

$$\sigma_\theta=\sigma_0\frac{(\nu-1)+(1+\nu)\left(r/R_2\right)^2}{(\nu-1)-(1+\nu)\left(R_1/R_2\right)^2}. \tag{17}$$

Based on (16), the wash material suffers pressure along radial directions; the pressure magnitude decreases towards the outside radius. On the other hand, in the azimuth (circumferential: $r = const$, $0 \le \theta \le 2\pi$) direction the material suffers stretching stress $\sigma_0(r)$. This is illustrated in Fig. 2 for various materials: steel ν = 0.3; copper ν = 0.25; rubber ν = 0.5.

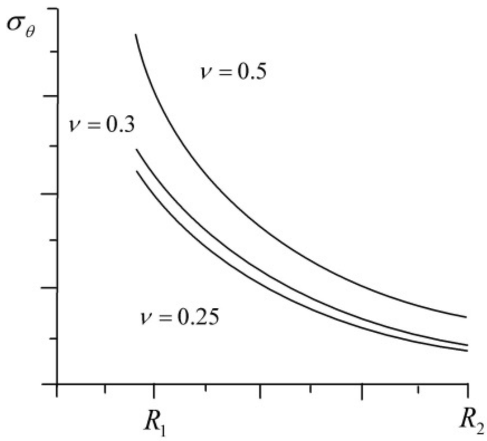

Fig. 2. Dependence of stretching stress on the radius.

3. Dynamic problem

Shear self-oscillations of the wash are described by a non-linear boundary value problem (12)–(14). Near the tipping point, where $M_{fr} = 0$, the moment of friction can be represented by a cubic polynomial of degrees of $\partial u / \partial t$ and has a dropping section [3]

$$M_{fr}\left(\omega R_1 t - \frac{\partial u_\theta}{\partial t}\right) = M_0 - M_1 u_{\theta t} + M_3 u_{\theta t}^3 . \tag{18}$$

Let us introduce new dimensionless variables $t_H = \sqrt{\mu / \rho} \cdot t / R$, $r_H = r / R_1$, $1 \le r \le R_2 / R_1$ and a new required function $\psi(r, t)$

$$u_\theta = \psi(r,t) - \frac{M_0 R_1^2}{2\mu R_2^2} \frac{r^2 - R_2^2}{r} . \tag{19}$$

The last item above describes static distribution of shear deflections $\frac{M_0 R_1^2}{2\mu R_2^2} \frac{r^2 - R_2^2}{r}$,

which are caused by a constant component in the moment of friction; $\psi(r, t)$ describes shear oscillations about this equilibrium

$$\psi_{tt} = \frac{\partial^2}{\partial r}\left(\psi_r + \psi / r\right) + \varepsilon \frac{\partial^3}{\partial t \partial r^2}\left(\psi_r + \psi / r\right), \tag{20}$$

$$\psi\big|_{r=R} = 0, \tag{21}$$

$$\left[\left(\psi_r + \psi / r\right) = \left(\alpha \psi_t - \beta \psi_t^3\right)\right]_{r=1}, \tag{22}$$

where $\varepsilon = hR_1 / \sqrt{\mu / \rho}$, $\alpha = M_1 / \sqrt{\mu / \rho}$, $\beta = M_3 \sqrt{\mu / \rho}$.

The solution (20)–(22) will be sought as an eigen-function series of a corresponding linear conservative problem.

4. Eigen functions and eigen values of the dynamic problem

Let us elaborate on the analysis of eigen functions and eigen values of the boundary value problem without considering the internal friction and frictional interaction ($\varepsilon = 0$, $\alpha = \beta = 0$). As is shown below, non-conservative items at a first approximation do not affect the real part of proper frequencies of the wash oscillations.

Eigen functions and eigen values for (20)–(22) are a solution to the Sturm-Liouville problem

$$\begin{gathered} r^2 X''(r) + rX'(r) + \left(\lambda^2 r^2 - 1\right) X(r) = 0 \\ X'(1) - X(1) = 0 \end{gathered} . \tag{23}$$

The general solution (23) is a linear superposition of the first-order Bessel function of the first and second kind $J_1(\lambda r)$, $Y_1(\lambda r)$

$$X(r) = C_1 J_1(\lambda r) + C_2 J_2(\lambda r). \tag{24}$$

The non-trivial solution (24) satisfying the boundary conditions results in the following spectral equation

$$\lambda = \frac{J_1(\lambda R)Y_1(\lambda) - Y_1(\lambda R)J_1(\lambda)}{J_1(\lambda R)Y_1'(\lambda) - Y_1(\lambda R)J_1'(\lambda)}. \tag{25}$$

The analytical solution to this equation is unknown. Table 1 lists magnitudes of the first five eigen values for various inside/outside radius ratios of the wash $\Delta R = R_2 / R_1$.

Table 1. Eigen values of the spectral equation.

$\Delta R \setminus \lambda_i$	λ_1	λ_2	λ_3	λ_4	λ_5
0.1	2.4	5.2	8.4	11.1	14.3
0.5	2.2	4.5	7.0	9.1	14.9
1.0	1.8	3.6	8.1	9.9	14.4
5.0	0.2	0.5	1.7	2.2	2.7
10.0	0.1	0.3	0.6	0.9	1.2

5. Energy estimation of the self-oscillation spectrum width

As is known, internal friction leads to frequency dependent elastic wave attenuation $\sim\omega^2$ [2]. Therefore, the self-oscillation spectrum may include only spectral components for which energy loss due to internal friction (oscillation decrement) is less than energy inflow due to "negative" friction work in the frictional contact (oscillation increment). System oscillations at frequencies that do not meet this condition decay naturally and can be disregarded in further analysis. The system self-excitation conditions can be found through solving the linearized problem (19)–(21) at $\beta = 0$

$$\psi_{tt} = \frac{\partial}{\partial r}(\psi_r + \psi / r) + \varepsilon \frac{\partial^2}{\partial r \partial t}(\psi_r + \psi / r), \tag{26}$$

$$\psi\big|_{r=R} = 0, \tag{27}$$

$$\left[\psi_r - \psi / r = \alpha \psi_t\right]_{r=1}. \tag{28}$$

6. System increment

To determine the increment of the system contributing to the energy of system oscillation caused by friction motion, a balance equation can be used. By multiplying (26) by ψ_t and grouping items appropriately, an energy transfer equation is obtained (Umov equation)

$$\frac{\partial W}{\partial t} = -\frac{\partial S}{\partial r} + \varepsilon r \psi_t (\psi_r + \psi / r), \tag{29}$$

where $W = \frac{r}{2}\left(\psi_t^2 + (\psi_r - \psi / r)^2\right)$ is the system's energy density; $S = r\psi_t(\psi_r - \psi / r)$ is the flow density.

Let us integrate (29) over the space variable "r" with provision for the boundary conditions (27) (28). The result will be the equation of change in total energy of the system

$$\frac{d}{dt} W = S\big|_{r=1} + \varepsilon \int_1^R r \psi_t (\psi_r + \psi / r)\, dr. \tag{30}$$

The first item in the right side of the energy balance equation (30) describes energy inflow due to "negative" friction work at the boundary $r = 1$; the second item details energy loss in the wash material. When determining system energy change due to dry friction at the boundary, set $\varepsilon = 0$. In this case, it follows from (30) that

$$S\Big|_{r=1} = \alpha\psi_t^2\Big|_{r=1}$$

or

$$\frac{d}{dt}W(t) = \alpha W(t)\,. \tag{31}$$

According to (31), the system energy grows in line with the exponential law $W \sim e^{\alpha t}$, irrespective of the oscillation frequency, the exponent indicator α is the energy increment of oscillations, whereas the amplitude increment is equal to $\alpha / 2$ correspondingly.

7. System decrement

When determining attenuation decrement responsible for frequency-dependent energy consumption by the wash material, disregard energy sources at the boundary (i.e., set $\alpha = 0$). Substitute the solution of the following to (25)

$$\psi = e^{i\omega t}X(\lambda r)\,, \tag{32}$$

where $X(\lambda, r)$, λ is the eigen function and eigen value respectively, obeying (22).

Simple manipulation results in a complex equation for determining oscillation frequency ω

$$\omega^2 = \lambda^2 + i\varepsilon\omega\lambda^2\,. \tag{33}$$

Its solution can be found by means of an expansion in powers of ε

$$\omega = \omega_1 + \varepsilon\omega_2 + \ldots \tag{34}$$

When limited to the order ε from (33), (34), the result is

$$\omega_1^2 = \lambda^2\,;\ \omega_2 = +i\lambda^2 / 2$$

or

$$\omega = \lambda + i\varepsilon\lambda^2 / 2\,. \tag{35}$$

It follows that a dissipative item in first approximation does not affect the real part of the frequency. The imaginary part is the attenuation decrement $\varepsilon\lambda^2/2$, while the wave amplitude (32) decreases as $e^{-\varepsilon\lambda^2 t/2}$.

The exceedance of the system increment over the decrement

$$\alpha \ge \varepsilon\lambda^2\,. \tag{36}$$

Is the system's self-excitation condition; also, self-oscillations will only have the spectral components λ_i complying to (36).

8. Determining amplitude of stationary oscillations

The amplitude of stationary self-oscillations is determined by solving the initial equation (19) obeying the boundary conditions (20) and (21). In a self-oscillating system, periodic movements only exist with specific amplitudes that correspond to the equality of energy inflow due to "negative" friction work and non-linear dissipation losses in the friction motion. As is known, systems of cubic non-linearity are characterized by a soft mode of self-oscillation excitation. Only one stationary amplitude can exist in this mode. Following the approximative method detailed in [5], look for a solution in the form of

$$\psi = \psi_0 + \varepsilon\psi_1 + \varepsilon^2\psi_2 + \ldots, \tag{37}$$

where ψ_0 is one of solutions for a conservative system obeying the equation (19), when $\varepsilon = 0$ and boundary conditions correspond to (22)

$$\psi_0 = A\left(J_1(\lambda r) - \frac{J_1(\lambda r)}{Y_1(\lambda r)} Y_1(\lambda r) \right)\sin\lambda t\ . \tag{38}$$

The amplitude A must be chosen close to the amplitude of the boundary cycle. In this case the small correction $\varepsilon\psi_1$ and terms corresponding to higher approximations, must not increase with time (that is, secular items are absent). By substituting (38) to (19)–(21), we obtain the following for the first correction

$$\frac{\partial^2\psi_1}{\partial t^2} - \frac{\partial}{\partial r}(\psi_{1r} + \psi_1 / r) = -\varepsilon\lambda^3 A\left(J_1''(\lambda r) - \frac{J_1(\lambda r)}{Y_1(\lambda r)} Y_1''(\lambda r) \right)\cos\lambda t\ , \tag{39}$$

$$\psi\big|_{r=R} = 0\ , \tag{40}$$

$$\left[\psi_{1r} + \psi_1 / r\right]_{r=1} = -\varepsilon\lambda A X(1)\left(\alpha\cos\lambda t - \beta\lambda^2 A^2 X^2(1)\cos^3\lambda t\right), \tag{41}$$

where $X(1)$ are the values of the eigen function when $r = 1$.

Disturbance equations (39)–(41) are written as equations describing the system dynamics affected by external periodic forces. Expressions for these forces include components with frequencies λ and 3λ. Since the frequency spectrum is inequivalent according to Table 1, the frequency dependent force component is not resonant and does not create movement of noticeable amplitude in the system; therefore, this force component can be neglected. In this case, the boundary condition (41) will look as follows

$$\left[\psi_{1r} + \psi_1 / r\right]_{r=1} = -\lambda A X(1)\left(\alpha - 3/4\beta\lambda^2 A^2 X^2(1)\right)\cos\lambda t\ , \tag{42}$$

Therefore, the conservative system with a respective frequency is affected by resonance frequency force that introduces secular items in the solution (37). In this case, stationary (limited) solution is only possible on condition of orthogonality of ψ_0 and ψ_1 (since ψ_0 behaves like $\sin(\lambda t)$, consequently, ψ_1 must behave like $\cos(\lambda t)$).

Suppose

$$\psi_1 = V(r)\cos\lambda t\ , \tag{43}$$

substituting (43) to (39) gives

$$\frac{\partial}{\partial r}\left(V_r + V / r\right) + \lambda^2 V = A\lambda^3 X^{//}\left(r\right). \tag{44}$$

A particular solution of the nonhomogeneous equation (44) looks as follows

$$V\left(r\right) = A\lambda \frac{r}{2} X\left(r\right), \tag{45}$$

where the function $X(r)$ obeys (40).
Substitute (43) to (42) with regard for (45) to get the following expression

$$\frac{d}{dr}\left(\frac{r}{2} X\left(r\right)\right)_{r=1} - \frac{X(1)}{2} = -X\left(1\right)\left(\alpha - 3/4\beta\lambda^2 A^2 X^2\left(1\right)\right), \tag{46}$$

which is an algebraic equation for defining the amplitude of frictional self-oscillations of the wash

$$A^2 = \frac{4\alpha}{3\lambda^2 \beta X^2\left(1\right)}, \tag{47}$$

Going back to (38), the law of shear single-frequency self-oscillations of the wash looks as follows

$$\psi\left(r,t\right) \approx \psi_0 + \psi_1 = \sqrt{\frac{4\alpha}{3\lambda^2 \beta X^2\left(1\right)}}\left(\sin\lambda t + \varepsilon \frac{\lambda r}{2}\cos\lambda t\right)\left(J_1\left(\lambda r\right) - \frac{J_1\left(\lambda R\right)}{Y_1\left(\lambda R\right)} Y_1\left(\lambda r\right)\right), \tag{48}$$

where λ is one of eigen values obeying the self-excitation condition (36). As seen from (48), the self-oscillation amplitude decreases in inverse proportion to frequency.

Since fluctuation disturbance is present in any real system, ultimately self-oscillation described in (48) will settle on every frequency that meets the excitation condition. Since, due to non-equidistant spectrum, these oscillations do not interact with one another, (48) can be used to describe the settled multi-frequency mode of the wash self-oscillations

$$\psi\left(r,t\right) = \sqrt{\frac{4\alpha}{3\beta}} \sum_{i:\alpha \geq \varepsilon\lambda i} \frac{1}{\lambda_i X_i\left(1\right)}\left(\sin\lambda_i t + \varepsilon \frac{\lambda_i r}{2}\cos\lambda_i t\right) X_i\left(r\right), \tag{49}$$

where λ_i is the i-th eigen function (23).
Issues of stability of rotor systems studied in [16-19].

9. Conclusion

In summary, the function $\psi(r, t)$ that details shear oscillations near the state of equilibrium also describes processes related to the bearing outer race of a machine in rotary mode. This allows evaluating pulsations of the rotary moment at the machine shaft. With regard to dynamoelectric systems, such pulsations affect dynamoelectric and electro-magnetic processes both in electric motors and generator sets [20, 21].

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